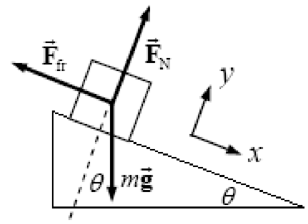
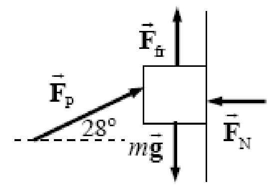
6. (*a*) Here is a free-body diagram for the box at rest on the plane. The force of friction is a STATIC frictional force, since the box is at rest.
(*b*) If the box were sliding down the plane, the only change is that the force of friction would be a KINETIC frictional force.
(*c*) If the box were sliding up the plane, the force of friction would be a KINETIC frictional force, and it would point down the plane, in the opposite direction to that shown in the diagram.
Notice that the angle is not used in this solution.



16. Consider a free-body diagram for the box, showing force on the box. When $F_P = 23\,N$, the block does not move. Thus in that case, the force of friction is static friction, and must be at its maximum value, given by $F_{fr} = \mu_s F_N$. Write Newton's second law in both the x and y directions. The net force in each case must be 0, since the block is at rest.



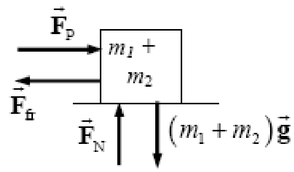
$$\sum F_x = F_P \cos\theta - F_N = 0 \quad \rightarrow \quad F_N = F_P \cos\theta$$

$$\sum F_y = F_{fr} + F_P \sin\theta - mg = 0 \quad \rightarrow \quad F_{fr} + F_P \sin\theta = mg$$

$$\mu_s F_N + F_P \sin\theta = mg \quad \rightarrow \quad \mu_s F_P \cos\theta + F_P \sin\theta = mg$$

$$m = \frac{F_P}{g}(\mu_s \cos\theta + \sin\theta) = \frac{23\,N}{9.80\,m/s^2}\left(0.40\cos 28^\circ + \sin 28^\circ\right) = \boxed{1.9\,kg}$$

17. (*a*) Since the two blocks are in contact, they can be treated as a single object as long as no information is needed about internal forces (like the force of one block pushing on the other block). Since there is no motion in the vertical direction, it is apparent that $F_N = (m_1 + m_2)g$, and so $F_{fr} = \mu_k F_N = \mu_k (m_1 + m_2) g$. Write Newton's second law for the horizontal direction.

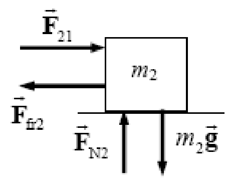


$$\sum F_x = F_P - F_{fr} = (m_1 + m_2) a \quad \rightarrow$$

$$a = \frac{F_P - F_{fr}}{m_1 + m_2} = \frac{F_P - \mu_k (m_1 + m_2) g}{m_1 + m_2} = \frac{650\,\text{N} - (0.18)(190\,\text{kg})(9.80\,\text{m/s}^2)}{190\,\text{kg}}$$

$$= 1.657\,\text{m/s}^2 \approx \boxed{1.7\,\text{m/s}^2}$$

(*b*) To solve for the contact forces between the blocks, an individual block must be analyzed. Look at the free-body diagram for the second block. $\vec{\mathbf{F}}_{21}$ is the force of the first block pushing on the second block. Again, it is apparent that $F_{N2} = m_2 g$ and so $F_{fr2} = \mu_k F_{N2} = \mu_k m_2 g$. Write Newton's second law for the horizontal direction.

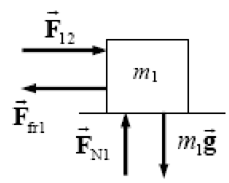


$$\sum F_x = F_{21} - F_{fr2} = m_2 a \quad \rightarrow$$

$$F_{21} = \mu_k m_2 g + m_2 a = (0.18)(125\,\text{kg})(9.80\,\text{m/s}^2) + (125\,\text{kg})(1.657\,\text{m/s}^2) = \boxed{430\,\text{N}}$$

By Newton's third law, there will also be a 430 N force to the left on block # 1 due to block # 2.

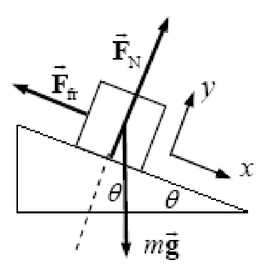
(*c*) If the crates are reversed, the acceleration of the system will remain the same – the analysis from part (*a*) still applies. We can also repeat the analysis from part (*b*) to find the force of one block on the other, if we simply change m_1 to m_2 in the free-body diagram and the resulting equations.



$$a = \boxed{1.7\,\text{m/s}^2} \;;\; \sum F_x = F_{12} - F_{fr1} = m_1 a \quad \rightarrow$$

$$F_{12} = \mu_k m_1 g + m_1 a = (0.18)(65\,\text{kg})(9.80\,\text{m/s}^2) + (65\,\text{kg})(1.657\,\text{m/s}^2) = \boxed{220\,\text{N}}$$

18. (*a*) Consider the free-body diagram for the crate on the surface. There is no motion in the *y* direction and thus no acceleration in the *y* direction. Write Newton's second law for both directions.



$$\sum F_y = F_N - mg\cos\theta = 0 \quad \rightarrow \quad F_N = mg\cos\theta$$

$$\sum F_x = mg\sin\theta - F_{fr} = ma$$

$$ma = mg\sin\theta - \mu_k F_N = mg\sin\theta - \mu_k mg\cos\theta$$

$$a = g(\sin\theta - \mu_k\cos\theta)$$

$$= (9.80\,\text{m/s}^2)(\sin 25.0^\circ - 0.19\cos 25.0^\circ) = 2.454\,\text{m/s}^2 \approx \boxed{2.5\,\text{m/s}^2}$$

(*b*) Now use Eq. 2-12c, with an initial velocity of 0, to find the final velocity.

$$v^2 - v_0^2 = 2a(x - x_0) \quad \rightarrow \quad v = \sqrt{2a(x - x_0)} = \sqrt{2(2.454\,\text{m/s}^2)(8.15\,\text{m})} = \boxed{6.3\,\text{m/s}}$$

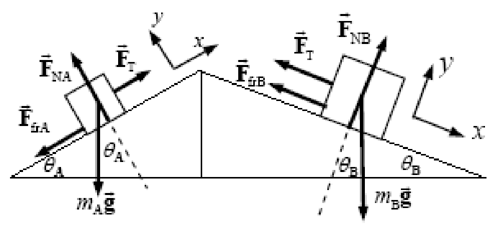
23. (*a*) For m_B to not move, the tension must be equal to $m_B g$, and so $m_B g = F_T$. For m_A to not move, the tension must be equal to the force of static friction, and so $F_S = F_T$. Note that the normal force on m_A is equal to its weight. Use these relationships to solve for m_A.

$$m_B g = F_T = F_s \le \mu_s m_A g \quad \rightarrow \quad m_A \ge \frac{m_B}{\mu_s} = \frac{2.0\,\text{kg}}{0.40} = 5.0\ \text{kg} \quad \rightarrow \quad m_A \ge \boxed{5.0\ \text{kg}}$$

(*b*) For m_B to move with constant velocity, the tension must be equal to $m_B g$. For m_A to move with constant velocity, the tension must be equal to the force of kinetic friction. Note that the normal force on m_A is equal to its weight. Use these relationships to solve for m_A.

$$m_B g = F_k = \mu_k m_A g \quad \rightarrow \quad m_A = \frac{m_B}{\mu_k} = \frac{2.0\,\text{kg}}{0.30} = \boxed{6.7\,\text{kg}}$$

28. We define the positive x direction to be the direction of motion for each block. See the free-body diagrams. Write Newton's second law in both dimensions for both objects. Add the two x-equations to find the acceleration.



Block A:

$$\sum F_{yA} = F_{NA} - m_A g \cos\theta_A = 0 \ \rightarrow \ F_{NA} = m_A g \cos\theta_A$$

$$\sum F_{xA} = F_T - m_A g \sin\theta - F_{frA} = m_A a$$

Block B:

$$\sum F_{yB} = F_{NB} - m_B g \cos\theta_B = 0 \ \rightarrow \ F_{NB} = m_B g \cos\theta_B$$

$$\sum F_{xB} = m_B g \sin\theta - F_{frB} - F_T = m_B a$$

Add the final equations together from both analyses and solve for the acceleration, noting that in both cases the friction force is found as $F_{fr} = \mu F_N$.

$$m_A a = F_T - m_A g \sin\theta_A - \mu_A m_A g \cos\theta_A \quad ; \quad m_B a = m_B g \sin\theta_B - \mu_B m_B g \cos\theta_B - F_T$$

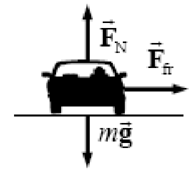
$$m_A a + m_B a = F_T - m_A g \sin\theta_A - \mu_A m_A g \cos\theta_A + m_B g \sin\theta_B - \mu_B m_B g \cos\theta_B - F_T \quad \rightarrow$$

$$a = g\left[\frac{-m_A(\sin\theta_A + \mu_A \cos\theta_A) + m_B(\sin\theta - \mu_B \cos\theta)}{(m_A + m_B)}\right]$$

$$= (9.80\,\text{m/s}^2)\left[\frac{-(2.0\,\text{kg})(\sin 51° + 0.30\cos 51°) + (5.0\,\text{kg})(\sin 21° - 0.30\cos 21°)}{(7.0\,\text{kg})}\right]$$

$$= \boxed{-2.2\,\text{m/s}^2}$$

34. A free-body diagram for the car at one instant of time is shown. In the diagram, the car is coming out of the paper at the reader, and the center of the circular path is to the right of the car, in the plane of the paper. If the car has its maximum speed, it would be on the verge of slipping, and the force of static friction would be at its maximum value. The vertical forces (gravity and normal force) are of the same magnitude, because the car is not accelerating vertically. We assume that the force of friction is the force causing the circular motion.

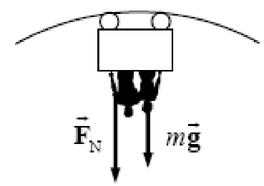


$$F_R = F_{fr} \rightarrow mv^2/r = \mu_s F_N = \mu_s mg \rightarrow$$

$$v = \sqrt{\mu_s rg} = \sqrt{(0.65)(80.0\,\text{m})(9.80\,\text{m/s}^2)} = 22.57\,\text{m/s} \approx \boxed{23\,\text{m/s}}$$

Notice that the result is $\boxed{\text{independent of the car's mass}}$.

40. At the top of a circle, a free-body diagram for the passengers would be as shown, assuming the passengers are upside down. Then the car's normal force would be pushing DOWN on the passengers, as shown in the diagram. We assume no safety devices are present. Choose the positive direction to be down, and write Newton's second law for the passengers.

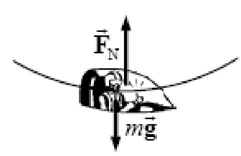


$$\sum F = F_N + mg = ma = mv^2/r \rightarrow F_N = m\left(v^2/r - g\right)$$

We see from this expression that for a high speed, the normal force is positive, meaning the passengers are in contact with the car. But as the speed decreases, the normal force also decreases. If the normal force becomes 0, the passengers are no longer in contact with the car – they are in free fall. The limiting condition is as follows.

$$v_{min}^2/r - g = 0 \rightarrow v_{min} = \sqrt{rg} = \sqrt{(9.80\,\text{m/s}^2)(7.6\,\text{m})} = \boxed{8.6\,\text{m/s}}$$

47. (*a*) See the free-body diagram for the pilot in the jet at the bottom of the loop. We have $a_R = v^2/r = 6g$.



$$v^2/r = 6.0g \rightarrow r = \frac{v^2}{6.0g} = \frac{\left[(1200\,\text{km/h})\left(\frac{1\,\text{m/s}}{3.6\,\text{km/h}}\right)\right]^2}{6.0(9.80\,\text{m/s}^2)} = \boxed{1900\,\text{m}}$$

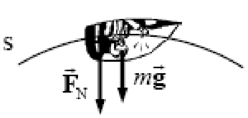
(*b*) The net force must be centripetal, to make the pilot go in a circle. Write Newton's second law for the vertical direction, with up as positive. The normal force is the apparent weight.

$$\sum F_R = F_N - mg = mv^2/r$$

The centripetal acceleration is to be $v^2/r = 6.0g$.

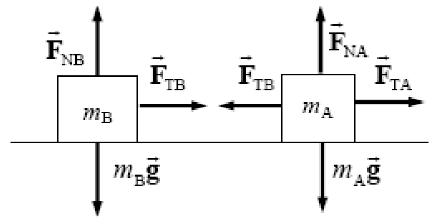
$$F_N = mg + mv^2/r = 7mg = 7(78\text{ kg})(9.80\,\text{m/s}^2) = 5350\,\text{N} = \boxed{5400\,\text{N}}$$

(*c*) See the free-body diagram for the pilot at the top of the loop. Notice that the normal force is down, because the pilot is upside down. Write Newton's second law in the vertical direction, with down as positive.



$$\sum F_R = F_N + mg = mv^2/r = 6mg \rightarrow F_N = 5mg = \boxed{3800\,\text{N}}$$

54. If the masses are in line and both have the same frequency of rotation, then they will always stay in line. Consider a free-body diagram for both masses, from a side view, at the instant that they are to the left of the post. Note that the same tension that pulls inward on mass 2 pulls outward on mass 1, by Newton's third law. Also notice that since there is no vertical acceleration, the normal force on each mass is equal to its weight. Write Newton's second law for the horizontal direction for both masses, noting that they are in uniform circular motion.



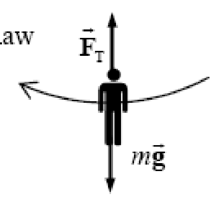
$$\sum F_{RA} = F_{TA} - F_{TB} = m_A a_A = m_A v_A^2/r_A \qquad \sum F_{RB} = F_{TB} = m_B a_B = m_B v_B^2/r_B$$

The speeds can be expressed in terms of the frequency as follows: $v = \left(f \dfrac{\text{rev}}{\text{sec}} \right)\left(\dfrac{2\pi r}{1\ \text{rev}} \right) = 2\pi r f$.

$$F_{TB} = m_B v_B^2/r_B = m_B \left(2\pi r_B f\right)^2/r_B = \boxed{4\pi^2 m_B r_B f^2}$$

$$F_{TA} = F_{TB} + m_A v_A^2/r_A = 4\pi m_B r_B f^2 + m_A \left(2\pi r_A f\right)^2/r_A = \boxed{4\pi^2 f^2 \left(m_A r_A + m_B r_B\right)}$$

55. A free-body diagram of Tarzan at the bottom of his swing is shown. The upward tension force is created by his pulling down on the vine. Write Newton's second law in the vertical direction. Since he is moving in a circle, his acceleration will be centripetal, and points upward when he is at the bottom.



$$\sum F = F_T - mg = ma = m v^2/r \quad \rightarrow \quad v = \sqrt{\frac{\left(F_T - mg\right) r}{m}}$$

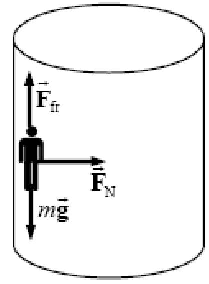
The maximum speed will be obtained with the maximum tension.

$$v_{max} = \sqrt{\frac{\left(\vec{F}_{T\,max} - mg\right) r}{m}} = \sqrt{\frac{\left(1350\ \text{N} - (78\,\text{kg})\left(9.80\,\text{m/s}^2\right)\right) 5.2\ \text{m}}{78\,\text{kg}}} = \boxed{6.2\,\text{m/s}}$$

80. Since mass m is dangling, the tension in the cord must be equal to the weight of mass m, and so $F_T = mg$. That same tension is in the other end of the cord, maintaining the circular motion of mass M, and so $F_T = F_R = Ma_R = M v^2/r$. Equate the expressions for tension and solve for the velocity.

$$M v^2/r = mg \quad \rightarrow \quad v = \boxed{\sqrt{mgR/M}}$$

82. Consider the free-body diagram for a person in the "Rotor-ride." $\vec{\mathbf{F}}_N$ is the normal force of contact between the rider and the wall, and $\vec{\mathbf{F}}_{fr}$ is the static frictional force between the back of the rider and the wall. Write Newton's second law for the vertical forces, noting that there is no vertical acceleration.



$$\sum F_y = F_{fr} - mg = 0 \quad \rightarrow \quad F_{fr} = mg$$

If we assume that the static friction force is a maximum, then

$$F_{fr} = \mu_s F_N = mg \quad \rightarrow \quad F_N = m\,g/\mu_s\,.$$

But the normal force must be the force causing the centripetal motion – it is the only force pointing to the center of rotation. Thus $F_R = F_N = m v^2/r$. Using $v = 2\pi r/T$, we have $F_N = \dfrac{4\pi^2 mr}{T^2}$. Equate the two expressions for the normal force and solve for the coefficient of friction. Note that since there are 0.50 rev per sec, the period is 2.0 sec.

$$F_N = \frac{4\pi^2 mr}{T^2} = \frac{mg}{\mu_s} \quad \rightarrow \quad \mu_s = \frac{gT^2}{4\pi^2 r} = \frac{\left(9.80\,\text{m/s}^2\right)\left(2.0\,\text{s}\right)^2}{4\pi^2\left(5.5\,\text{m}\right)} = \boxed{0.18}$$

Any larger value of the coefficient of friction would mean that the normal force could be smaller to achieve the same frictional force, and so the period could be longer or the cylinder radius smaller.

There is no force pushing outward on the riders. Rather, the wall pushes against the riders, so by Newton's third law, the riders push against the wall. This gives the sensation of being pressed into the wall.