$\boxed{1.}$ The potential energy of the spring is given by $U_{el} = \frac{1}{2}kx^2$ where x is the distance of stretching or compressing of the spring from its natural length.

$$x = \sqrt{\frac{2U_{el}}{k}} = \sqrt{\frac{2(35.0\,\text{J})}{82.0\,\text{N/m}}} = \boxed{0.924\,\text{m}}$$

16. (*a*) Since there are no dissipative forces present, the mechanical energy of the person–trampoline–Earth combination will be conserved. We take the level of the unstretched trampoline as the zero level for both elastic and gravitational potential energy. Call up the positive direction. Subscript 1 represents the jumper at the start of the jump, and subscript 2 represents the jumper upon arriving at the trampoline. There is no elastic potential energy involved in this part of the problem. We have $v_1 = 4.5\,\text{m/s}$, $y_1 = 2.0\,\text{m}$, and $y_2 = 0$. Solve for v_2, the speed upon arriving at the trampoline.

$$E_1 = E_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + 0 \rightarrow$$

$$v_2 = \pm\sqrt{v_1^2 + 2gy_1} = \pm\sqrt{(4.5\,\text{m/s})^2 + 2(9.80\,\text{m/s}^2)(2.0\,\text{m})} = \pm 7.710\,\text{m/s} \approx \boxed{7.7\,\text{m/s}}$$

The speed is the absolute value of v_2.

(*b*) Now let subscript 3 represent the jumper at the maximum stretch of the trampoline, and x represent the amount of stretch of the trampoline. We have $v_2 = -7.710\,\text{m/s}$, $y_2 = 0$, $x_2 = 0$, $v_3 = 0$, and $x_3 = y_3$. There is no elastic energy at position 2, but there is elastic energy at position 3. Also, the gravitational potential energy at position 3 is negative, and so $y_3 < 0$. A quadratic relationship results from the conservation of energy condition.

$$E_2 = E_3 \rightarrow \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}kx_2^2 = \frac{1}{2}mv_3^2 + mgy_3 + \frac{1}{2}kx_3^2 \rightarrow$$

$$\frac{1}{2}mv_2^2 + 0 + 0 = 0 + mgy_3 + \frac{1}{2}ky_3^2 \rightarrow \frac{1}{2}ky_3^2 + mgy_3 - \frac{1}{2}mv_2^2 = 0 \rightarrow$$

$$y_3 = \frac{-mg \pm \sqrt{m^2g^2 - 4\left(\frac{1}{2}k\right)\left(-\frac{1}{2}mv_2^2\right)}}{2\left(\frac{1}{2}k\right)} = \frac{-mg \pm \sqrt{m^2g^2 + kmv_2^2}}{k}$$

$$= \frac{-(72\,\text{kg})(9.80\,\text{m/s}^2) \pm \sqrt{(72\,\text{kg})^2(9.80\,\text{m/s}^2)^2 + (5.8\times10^4\,\text{N/m})(72\,\text{kg})(7.71\,\text{m/s})^2}}{(5.8\times10^4\,\text{N/m})}$$

$$= -0.284\,\text{m}\,,\ 0.260\,\text{m}$$

Since $y_3 < 0$, $y_3 = \boxed{-0.28\,\text{m}}$..

The first term under the quadratic is about 500 times smaller than the second term, indicating that the problem could have been approximated by not even including gravitational potential energy for the final position. If that approximation were made, the result would have been found by taking the negative result from the following solution.

$$E_2 = E_3 \rightarrow \frac{1}{2}mv_2^2 = \frac{1}{2}ky_3^2 \rightarrow y_3 = v_2\sqrt{\frac{m}{k}} = (7.71\,\text{m/s})\sqrt{\frac{72\,\text{kg}}{5.8\times10^4\,\text{N/m}}} = \pm 0.27\,\text{m}$$

20. Since there are no dissipative forces present, the mechanical energy of the roller coaster will be conserved. Subscript 1 represents the coaster at point 1, etc. The height of point 2 is the zero location for gravitational potential energy. We have $v_1 = 0$ and $y_1 = 32\,\text{m}$.

Point 2: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 \quad ; \; y_2 = 0 \;\rightarrow\; mgy_1 = \frac{1}{2}mv_2^2 \;\rightarrow$

$$v_2 = \sqrt{2gy_1} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(32\,\text{m}\right)} = \boxed{25\,\text{m/s}}$$

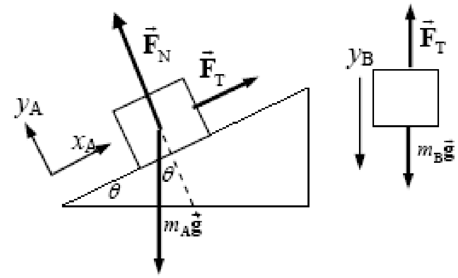
Point 3: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_3^2 + mgy_3 \quad ; \; y_3 = 26\,\text{m} \;\rightarrow\; mgy_1 = \frac{1}{2}mv_3^2 + mgy_3 \;\rightarrow$

$$v_3 = \sqrt{2g\left(y_1 - y_3\right)} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(6\,\text{m}\right)} = \boxed{11\,\text{m/s}}$$

Point 4: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_4^2 + mgy_4 \quad ; \; y_4 = 14\,\text{m} \;\rightarrow\; mgy_1 = \frac{1}{2}mv_4^2 + mgy_1 \;\rightarrow$

$$v_4 = \sqrt{2g\left(y_1 - y_4\right)} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(18\,\text{m}\right)} = \boxed{19\,\text{m/s}}$$

22. (*a*) Draw a free-body diagram for each block. Write Newton's second law for each block. Notice that the acceleration of block A in the y_A is 0 zero.



$$\sum F_{y1} = F_N - m_A g\cos\theta = 0 \quad\rightarrow\quad F_N = m_A g\cos\theta$$

$$\sum F_{x1} = F_T - m_A g\sin\theta = m_A a_{xA}$$

$$\sum F_{y2} = m_B g - F_T = m_B a_{yB} \quad\rightarrow\quad F_T = m_B\left(g + a_{yB}\right)$$

Since the blocks are connected by the cord, $a_{yB} = a_{xA} = a$. Substitute the expression for the tension force from the last equation into the x direction equation for block 1, and solve for the acceleration.

$$m_B\left(g + a\right) - m_A g\sin\theta = m_A a \quad\rightarrow\quad m_B g - m_A g\sin\theta = m_A a + m_B a$$

$$a = g\frac{\left(m_B - m_A\sin\theta\right)}{\left(m_A + m_B\right)} = \left(9.80\,\mathrm{m/s^2}\right)\frac{\left(5.0\,\mathrm{kg} - 4.0\,\mathrm{kg}\sin 32^\circ\right)}{9.0\,\mathrm{kg}} = \boxed{3.1\,\mathrm{m/s^2}}$$

(*b*) Find the final speed of m_B (which is also the final speed of m_A) using constant acceleration relationships.

$$v_f^2 = v_0^2 + 2a\Delta y \quad\rightarrow\quad v_f^2 = 2g\frac{\left(m_B - m_A\sin\theta\right)}{\left(m_A + m_B\right)}h \quad\rightarrow$$

$$v_f = \sqrt{2gh\frac{\left(m_B - m_A\sin\theta\right)}{\left(m_A + m_B\right)}} = \sqrt{2\left(9.80\,\mathrm{m/s^2}\right)\left(0.75\,\mathrm{m}\right)\frac{\left(5.0\,\mathrm{kg} - 4.0\,\mathrm{kg}\sin 32^\circ\right)}{9.0\,\mathrm{kg}}} = \boxed{2.2\,\mathrm{m/s}}$$

(*c*) Since there are no dissipative forces in the problem, the mechanical energy of the system is conserved. Subscript 1 represents the blocks at the release point, and subscript 2 represents the blocks when m_B reaches the floor. The ground is the zero location for gravitational potential energy for m_B, and the starting location for m_A is its zero location for gravitational potential energy. Since m_B falls a distance h, m_A moves a distance h along the plane, and so rises a distance $h\sin\theta$. The starting speed is 0.

$$E_1 = E_2 \quad\rightarrow\quad 0 + m_A gh = \tfrac{1}{2}\left(m_A + m_B\right)v_2^2 + m_B gh\sin\theta \quad\rightarrow$$

$$\boxed{v_2 = \sqrt{2gh\left(\frac{m_A - m_B\sin\theta}{m_A + m_B}\right)}}$$

This is the same expression found in part (*b*), and so gives the same numeric result.

42. (*a*) Use conservation of energy. Subscript 1 represents the block at the compressed location, and subscript 2 represents the block at its maximum position up the slope. The initial location of the block at the bottom of the plane is taken to be the zero location for gravitational potential energy ($y = 0$). The variable x will represent the amount of spring compression or stretch. We have $v_1 = 0$, $x_1 = 0.50\,\mathrm{m}$, $y_1 = 0$, $v_2 = 0$, and $x_2 = 0$. The distance the block moves up the plane is given by $d = \dfrac{y}{\sin\theta}$, so $y_2 = d\sin\theta$. Solve for d.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$\tfrac{1}{2}kx_1^2 = mgy_2 = mgd\sin\theta \quad \rightarrow \quad d = \frac{kx_1^2}{2mg\sin\theta} = \frac{(75\,\text{N/m})(0.50\,\text{m})^2}{2(2.0\,\text{kg})(9.80\,\text{m/s}^2)\sin 41^\circ} = \boxed{0.73\,\text{m}}$$

(*b*) Now the spring will be stretched at the turning point of the motion. The first half-meter of the block's motion returns the block to the equilibrium position of the spring. After that, the block beings to stretch the spring. Accordingly, we have the same conditions as before except that $x_2 = d - 0.5\,\text{m}$.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$\tfrac{1}{2}kx_1^2 = mgd\sin\theta + \tfrac{1}{2}k(d - 0.5\,\text{m})^2$$

This is a quadratic relation in d. Solving it gives $d = \boxed{0.66\,\text{m}}$.

~~(*c*) The block now moves $d = 0.50\,\text{m}$, and stops at the equilibrium point of the spring. Accordingly,~~

SKIP PART (c)

$$\Delta K + \Delta U \ldots \mu_k mgd\cos\theta \quad \rightarrow$$

$$\mu_k = \frac{-\tfrac{1}{2}k\ldots}{-mgd\cos\theta} = \frac{\ldots}{2mgd\cos\theta} - \tan\theta$$

$$= \frac{(75\,\text{N/m})(0.50\,\text{m})^2}{2(2.0\,\text{kg})(9.80\,\text{m/s}^2)(0.50\,\text{m})\cos 41^\circ} - \tan 41^\circ = \boxed{0.40}$$

$\boxed{85.}$ (*a*) The tension in the cord is perpendicular to the path at all times, and so the tension in the cord does not do any work on the ball. Thus only gravity does work on the ball, and so the mechanical energy of the ball is conserved. Subscript 1 represents the ball when it is horizontal and subscript 2 represents the ball at the lowest point on its path. The lowest point on the path is the zero location for potential energy $(y = 0)$. We have $v_1 = 0$, $y_1 = \ell$, and $y_2 = 0$. Solve for v_2.

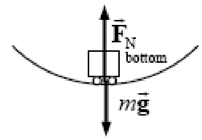
$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad mg\ell = \tfrac{1}{2}mv_2^2 \quad \rightarrow \quad v_2 = \boxed{\sqrt{2g\ell}}$$

(*b*) Use conservation of energy, to relate points 2 and 3. Point 2 is as described above. Subscript 3 represents the ball at the top of its circular path around the peg. The lowest point on the path is the zero location for potential energy $(y = 0)$. We have $v_2 = \sqrt{2g\ell}$, $y_2 = 0$, and $y_3 = 2(\ell - h) = 2(\ell - 0.80\ell) = 0.40\ell$. Solve for v_3.

$$E_2 = E_3 \quad \rightarrow \quad \tfrac{1}{2}mv_2^2 + mgy_2 = \tfrac{1}{2}mv_3^2 + mgy_3 \quad \rightarrow \quad \tfrac{1}{2}m(2g\ell) = \tfrac{1}{2}mv_3^2 + mg(0.40\ell) \quad \rightarrow$$

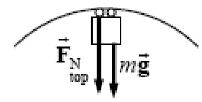
$$\boxed{v_3 = \sqrt{1.2g\ell}}$$

87. Consider the free-body diagram for the coaster at the bottom of the loop. The net force must be an upward centripetal force.



$$\sum F_{\text{bottom}} = F_{\substack{\mathrm{N}\\ \text{bottom}}} - mg = m v_{\text{bottom}}^2 / R \quad \rightarrow \quad F_{\substack{\mathrm{N}\\ \text{bottom}}} = mg + m v_{\text{bottom}}^2 / R$$

Now consider the force diagram at the top of the loop. Again, the net force must be centripetal, and so must be downward.



$$\sum F_{\text{top}} = F_{\substack{\mathrm{N}\\ \text{top}}} + mg = m v_{\text{top}}^2 / R \quad \rightarrow \quad F_{\substack{N\\ \text{top}}} = m v_{\text{top}}^2 / R - mg\,.$$

Assume that the speed at the top is large enough that $F_{\substack{N\\ \text{top}}} > 0$, and so $v_{\text{top}} > \sqrt{Rg}$.

Now apply the conservation of mechanical energy. Subscript 1 represents the coaster at the bottom of the loop, and subscript 2 represents the coaster at the top of the loop. The level of the bottom of the loop is the zero location for potential energy $(y = 0)$. We have $y_1 = 0$ and $y_2 = 2R$.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2} m v_1^2 + m g y_1 = \tfrac{1}{2} m v_2^2 + m g y_2 \quad \rightarrow \quad v_{\text{bottom}}^2 = v_{\text{top}}^2 + 4gR$$

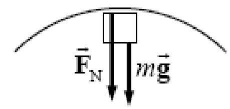
The difference in apparent weights is the difference in the normal forces.

$$F_{\substack{\mathrm{N}\\ \text{bottom}}} - F_{\substack{\mathrm{N}\\ \text{top}}} = \left(mg + m v_{\text{bottom}}^2 / R\right) - \left(m v_{\text{top}}^2 / R - mg\right) = 2mg + m\left(v_{\text{bottom}}^2 - v_{\text{top}}^2\right) / R$$

$$= 2mg + m\left(4gR\right)/R = \boxed{6mg}$$

Notice that the result does not depend on either v or R.

90. (*a*) Draw a free-body diagram for the block at the top of the curve. Since the block is moving in a circle, the net force is centripetal. Write Newton's second law for the block, with down as positive. If the block is to be on the verge of falling off the track, then $F_N = 0$.



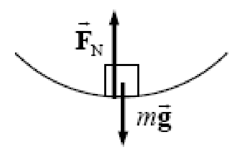
$$\sum F_R = F_N + mg = mv^2/r \quad \rightarrow \quad mg = mv_{top}^2/r \quad \rightarrow \quad v_{top} = \sqrt{gr}$$

Now use conservation of energy for the block. Since the track is frictionless, there are no non-conservative forces, and mechanical energy will be conserved. Subscript 1 represents the block at the release point, and subscript 2 represents the block at the top of the loop. The ground is the zero location for potential energy $(y = 0)$. We have $v_1 = 0,\ y_1 = h,\ v_2 = \sqrt{gr},$ and $y_2 = 2r$. Solve for h.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad 0 + mgh = \tfrac{1}{2}mgr + 2mgr \quad \rightarrow$$

$$h = \boxed{2.5r}$$

(*b*) See the free-body diagram for the block at the bottom of the loop. The net force is again centripetal, and must be upwards.



$$\sum F_R = F_N - mg = mv^2/r \quad \rightarrow \quad F_N = mg + mv_{bottom}^2/r$$

The speed at the bottom of the loop can be found from energy conservation, similar to what was done in part (*a*) above, by equating the energy at the release point (subscript 1) and the bottom of the loop (subscript 2). We now have $v_1 = 0,$ $y_1 = 2h = 5r,$ and $y_2 = 0$. Solve for v_2.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad 0 + 5mgr = \tfrac{1}{2}mv_{bottom}^2 + 0 \quad \rightarrow$$

$$v_{bottom}^2 = 10gr \quad \rightarrow \quad F_N = mg + mv_{bottom}^2/r = mg + 10mg = \boxed{11mg}$$

(*c*) Again we use the free body diagram for the top of the loop, but now the normal force does not vanish. We again use energy conservation, with $v_1 = 0,\ y_1 = 3r,$ and $y_2 = 0$. Solve for v_2.

$$\sum F_R = F_N + mg = mv^2/r \quad \rightarrow \quad F_N = mv_{top}^2/r - mg$$

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}mv_1^2 + mgy_1 = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \rightarrow \quad 0 + 3mgr = \tfrac{1}{2}mv_{top}^2 + 0 \quad \rightarrow$$

$$v_{top}^2 = 6gr \quad \rightarrow \quad F_N = mv_{top}^2/r - mg = 6mg - mg = \boxed{5mg}$$

(*d*) On the flat section, there is no centripetal force, and $F_N = \boxed{mg}$.