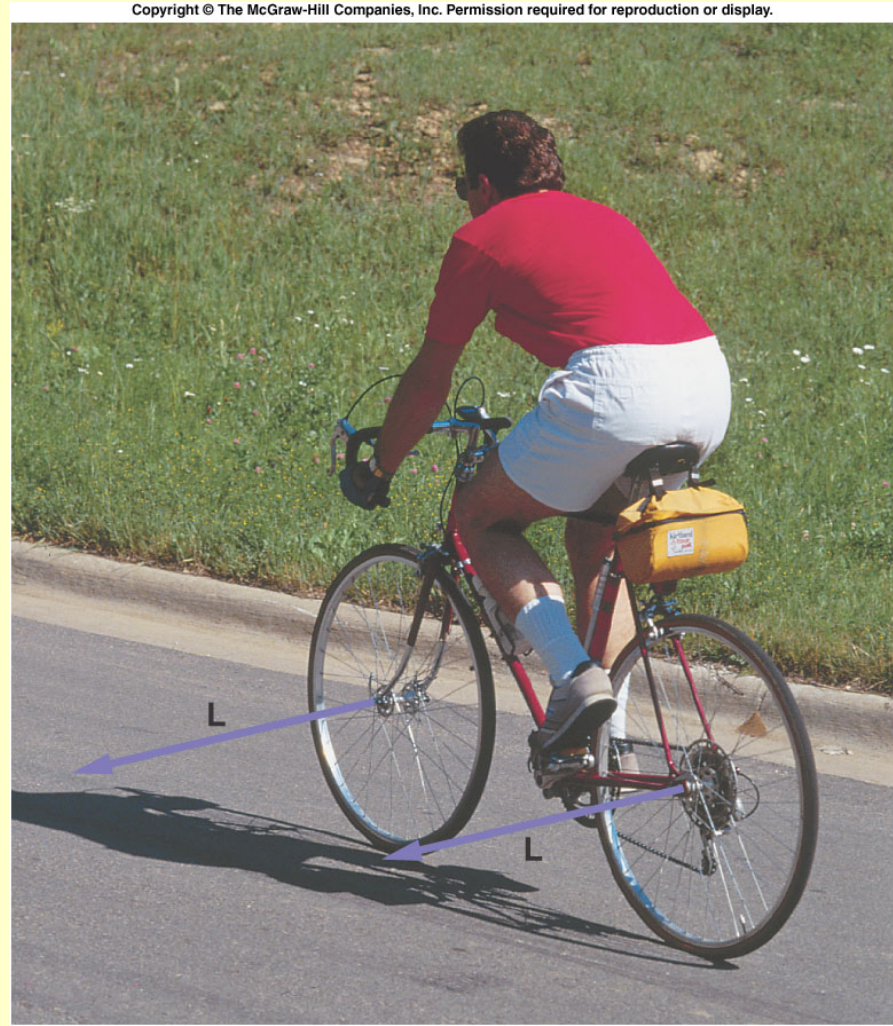
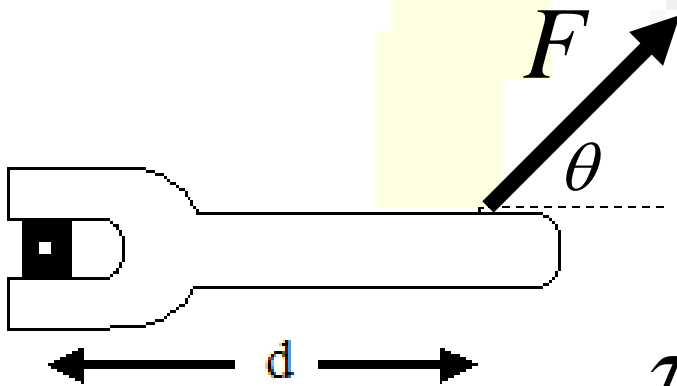


# Riding a bicycle and other amazing feats

Why does a bicycle remain upright when it is moving but promptly falls over when not moving?

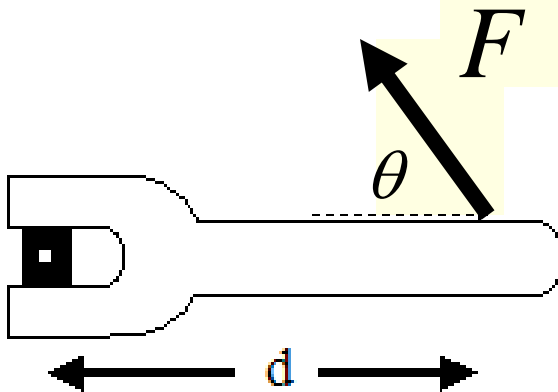


# TORQUE



$$\tau = Fd \sin \theta$$

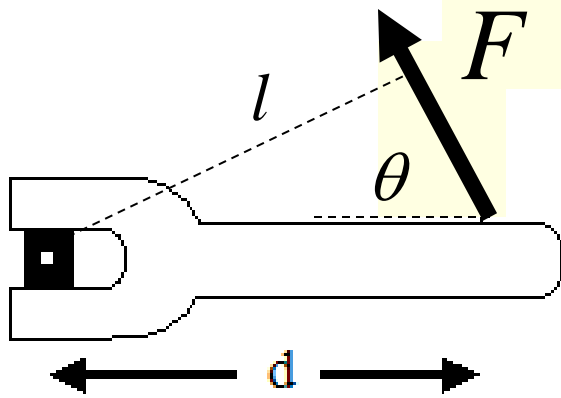
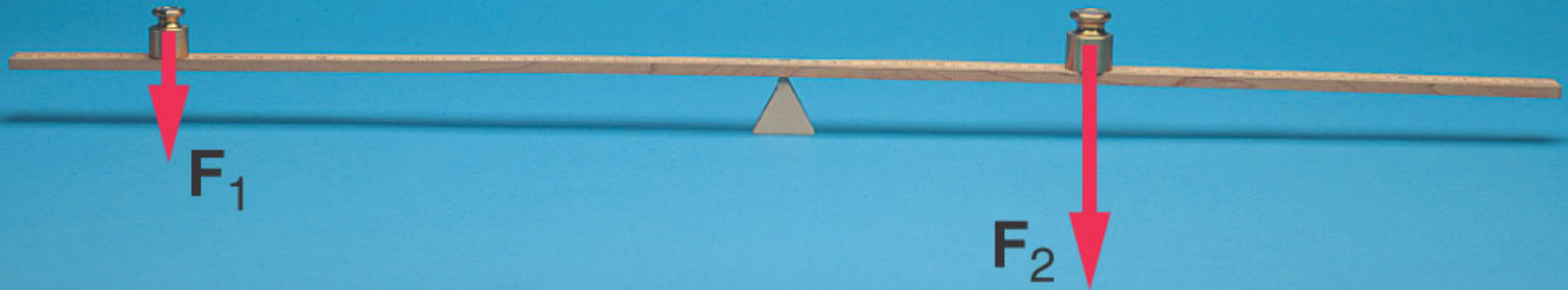
# TORQUE



$$\tau = Fd \sin \theta$$

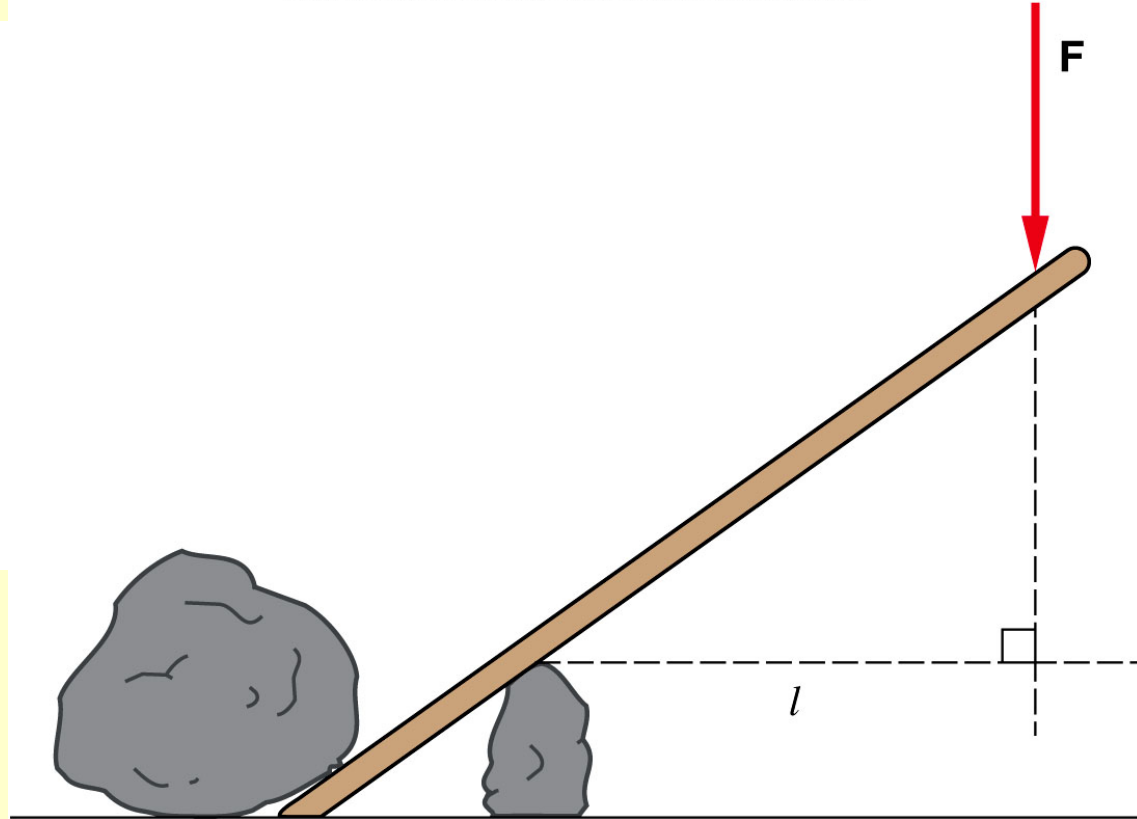
# Staplers





$$\tau = Fd \sin \theta$$

$$\tau = Fl$$



# Angular Variables

- $s = \theta r$
- $v = \omega r$
- $a = \alpha r$

.....

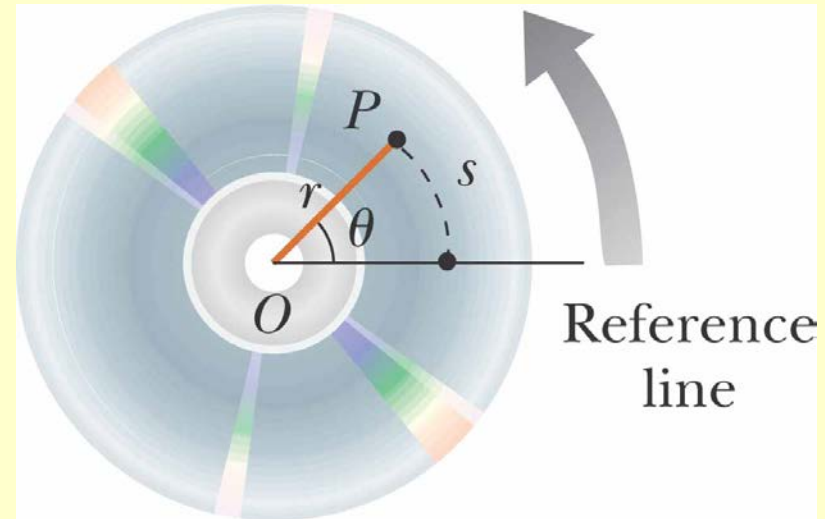
*Torque*

$$\tau = Fr$$

.....

*Angular momentum*

$$L = p r$$

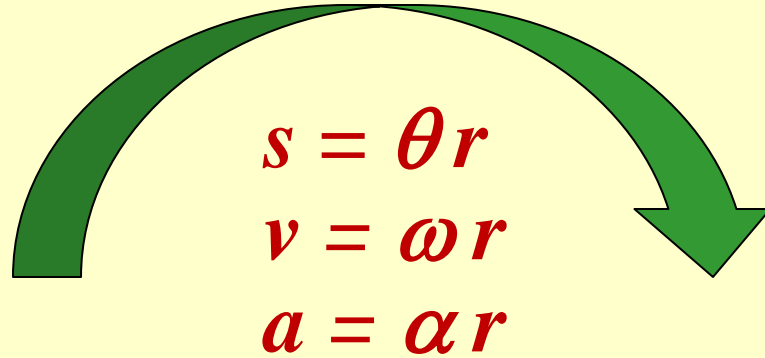


$$\tau = F r = m a r = m (\alpha r) r =$$

$$\tau = (m r^2) \alpha$$

$$\tau = I \alpha$$

where  $I = m r^2$



$$v = v_0 + at$$

$$\omega = \omega_0 + \alpha t$$

$$\Delta s = v_0 t + \frac{1}{2} at^2$$

$$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

# Conservation of Angular Momentum

- *Linear momentum* is conserved if the net external *force* acting on the system is zero.
- *Angular momentum* is conserved if the net external *torque* acting on the system is zero.

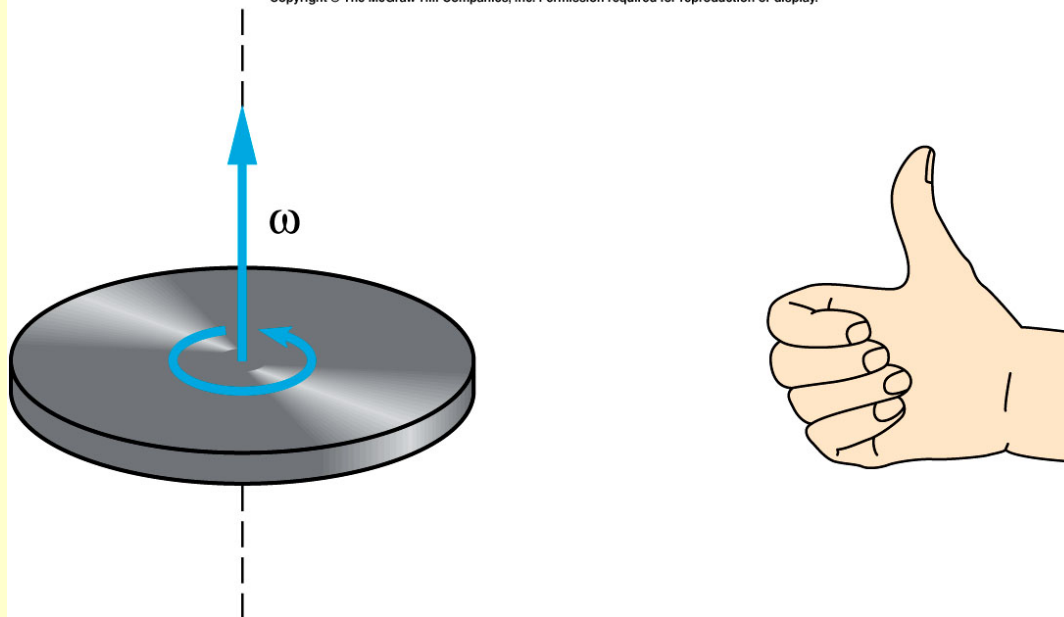
$$\begin{aligned} \text{Inertia } m : \quad & \mathbf{F}_{\text{net}} = m\mathbf{a} \\ & \mathbf{p} = m\mathbf{v} \\ \text{If } \mathbf{F}_{\text{net}} = 0, \quad & \mathbf{p} = \text{constant} \\ & KE = \frac{1}{2}mv^2 \end{aligned}$$

$$\begin{aligned} \text{Inertia } I : \quad & \tau_{\text{net}} = I\alpha \\ & L = I\omega = (mv)r \\ \text{If } \tau_{\text{net}} = 0, \quad & L = \text{constant} \\ & KE = \frac{1}{2}I\omega^2 \end{aligned}$$



# Angular Velocity

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



Directions:

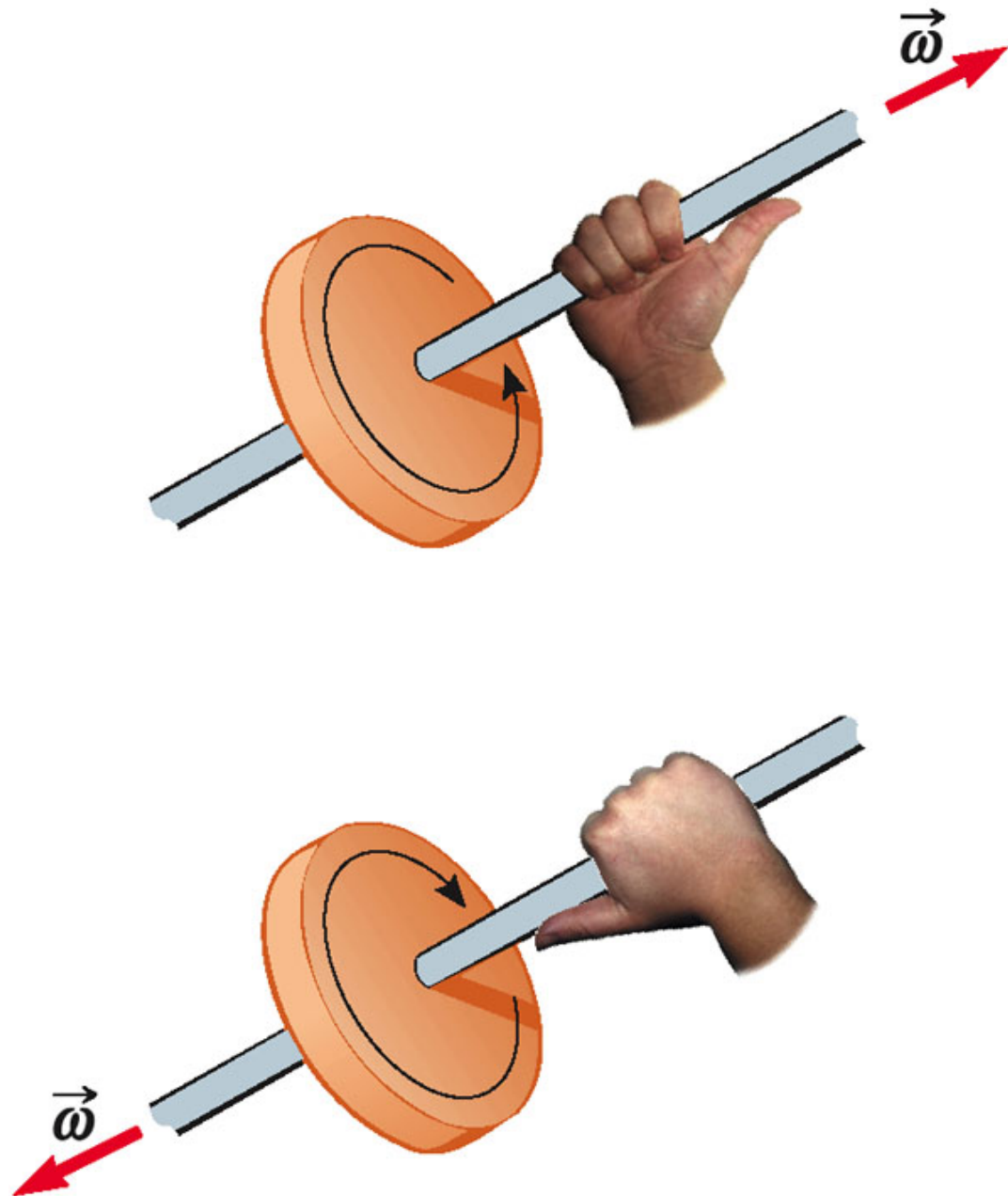
Angular  
Velocity

$\omega$

and

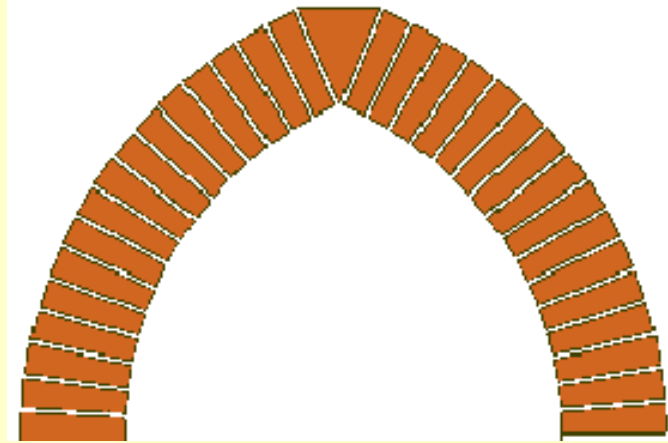
Momentum

$L$

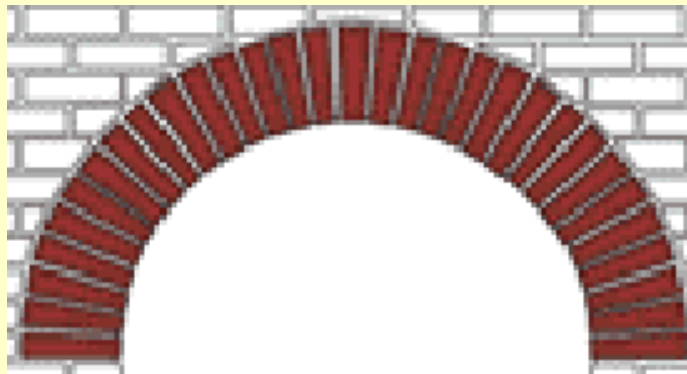




# Torque on Gothic Arches



Versus Roman Arches



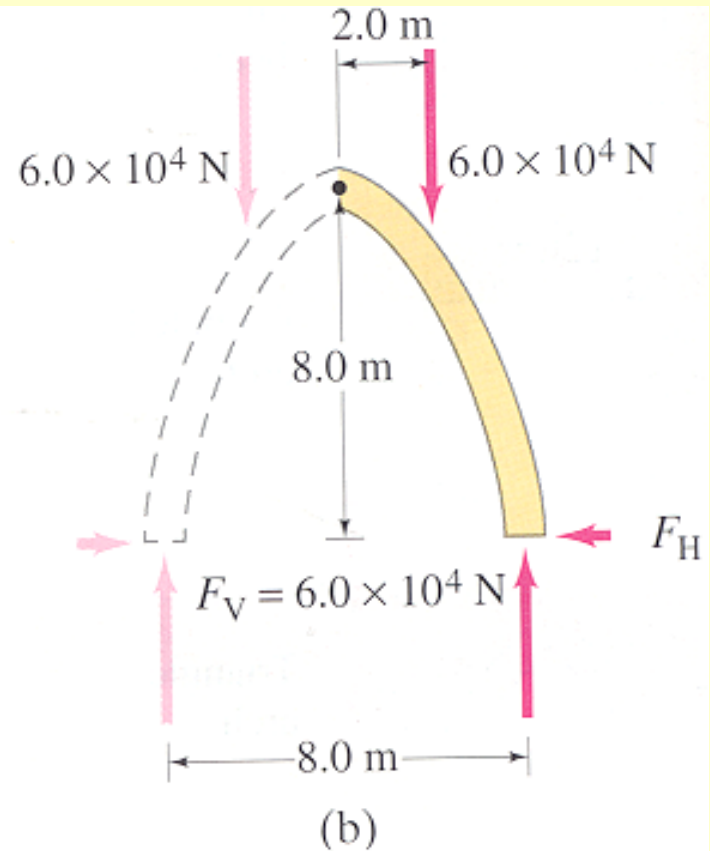
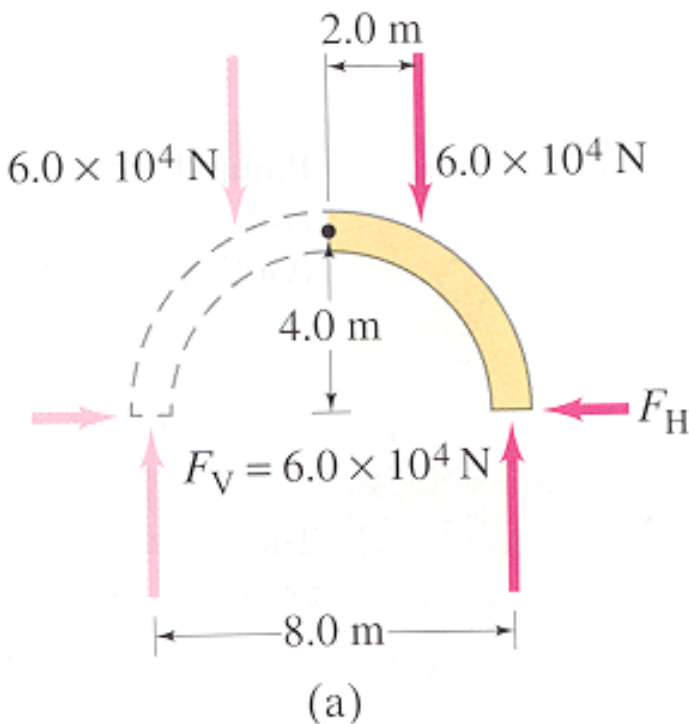
# Torque is reduced to half!

$$(4.0\text{m})(6.0 \times 10^4 \text{ N}) - (2.0\text{m})(6.0 \times 10^4 \text{ N}) - (4.0\text{m})(F_H) = 0$$

$$F_H = 3.0 \times 10^4 \text{ N}$$

$$(4.0\text{m})(6.0 \times 10^4 \text{ N}) - (2.0\text{m})(6.0 \times 10^4 \text{ N}) - (8.0\text{m})(F_H) = 0$$

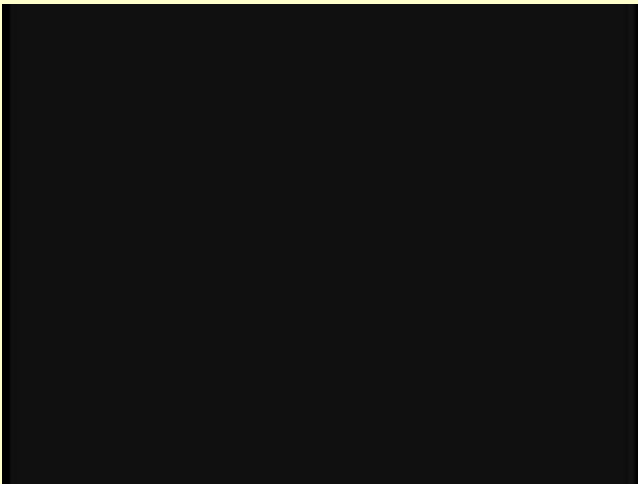
→  $F_H = 1.5 \times 10^4 \text{ N}$



**Pantheon  
in Rome  
118-35 A.D**



# Florence, Italy: Brunelleschi's Dome



# Pantheon in Rome 118-35 A.D



The interior is a perfect circle which diameter and height are exactly same, 43m.

The wall is 6.05m thick and on the lower level are seven niches with a pair of Corinthian columns.

The lower level and the second level are divided by the cornis in the ratio of a square root of 2 to 1.

Exterior walls are divided into two zone by the cornis but no correspondence with the height of the interior cornis. The hemispherical dome has the skylight oculus of 8.9m in diameter.

The second level is the re-design in 1747 which consists of a row of blind windows alternating with square designs.

Now one span of the second level was restored to the original state as seen in the whitish part of above image, that is six Corinthian columns with two niches.

You should notice that the design pattern of original second level is the same pattern of the lower level which is seen in the top image of this page. The real columns and pilasters of lower level are repeated again on the upper walls as graphic images.

This kind of design technique, the repeat and the superimpose are frequently used in high Renaissance, Mannerism, Baroque and of course in modern age.





# Cathedral of Notre Dame in Paris



# Leaning Tower of Pisa, Tuscany, Italy

What did go wrong?





# Understanding Torque and Moving Big Rocks: **THE MYSTERY of STONEHENGE**



<http://www.youtube.com/watch?v=1RRDzFROMx0>

# Rotational Inertia

